The Islamic University of Gaza **MATHA 1401 Final Exam.**

Department of Mathematics **First Semester**

6-1-2008 **Time:Two Hours**

Student name:	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**	**Total**
Student number:	25	19	18	18	20	100
Instructor name:						

Solve the following questions:-

Q1. (a) Find the derivative of the functions

(i) $y = (e^{2x} + 1)^{\tan x}$.

(ii) $y = \sec^{-1}(3^x) + \log_3(\sec x)$.

(b) Let $y = \int_0^{\sin x} t \sin^{-1} t \; dt, \; 0 \leq x \leq \frac{\pi}{2}$. show that $\frac{dy}{dx} = \frac{1}{2} x \sin 2x$.

(c) Find the following limits

(i) $\lim_{x \longrightarrow 1} \dfrac{1 - x + \ln x}{x^3 - 3x + 2}$.

(ii) $\lim_{x \longrightarrow \infty} \dfrac{\cos(x+1)}{e^x + 1}$.

Q2. (a) Simplify the expression $\cos(\tan^{-1} \frac{3x}{2})$.

(b) Solve for x : $\log_4(x+1) - \dfrac{1}{\log_x 4} = \dfrac{1}{2}$.

(c) Find the area of the region between the graph of the curves

$$y = (x-1)^2 \text{ and } y = x + 1 \text{ on the interval } [-1, 3].$$

Q3. Evaluate the following integrals:-

(a) $\int \sec^3 x \tan x dx.$

(b) $\int \frac{x+2}{x^2+4} dx.$

(c) $\displaystyle\int_1^{e^\pi} \frac{\cos^2(\ln x)}{x} dx$

Q4. Consider the function $f(x) = x^4 - 6x^2$.

(a) Find the x and y intercepts of f.

(b) Find the intervals for which f is increasing and decreasing.

(c) Find the relative extreme values of f.

(d) Find the intervals for which f is concave up and concave down.

(e) Find the inflection points of f.

(f) Sketch the graph of f.

Q5. Circle the appropriate answer for each of the following:

(1) $\cosh x$ equals $(\dfrac{e^x - e^{-x}}{2}\ ,\quad \dfrac{e^x + e^{-x}}{2}\ ,\quad \dfrac{2}{e^x - e^{-x}}\ ,\quad \dfrac{2}{e^x + e^{-x}})$.

(2) $\lim\limits_{x \longrightarrow -\infty} e^{-x}$ equals $(0,\quad \infty,\quad -\infty,\quad$ non of the above).

(3) If $f(x) = \ln x$ then $f(xe)$ equals $(f(x),\quad ef(x),\quad e + f(x),\quad$ non of the above).

(4) $\cos^{-1}(\cos(\frac{-\pi}{4}))$ equals $(\frac{-\pi}{4},\quad \frac{\pi}{4},\quad \frac{3\pi}{4},\quad$ does not exists).

(5) The domain of the function $f(x) = \sin^{-1} x + \ln x$ is $([-1, 1],\ (0, \infty),\ (0, 1],\ [-1, \infty))$.

(6) If $f'(x) = 3$ for all x then f is (increasing, integrable, not constant function, all of above).

(7) If $f'(c) = 0$, then at $x = c$, f must
(has a relative maximum value, has an inflection point, satisfy Rolle's theorem, be continuous).

(8) If $x < 0$ then $(|-x| = -|x|,\quad |-x| = x,\quad |-x| = -x,\quad$ non of the above).

(9) If $f(x) = [x]$ the greatest integer function then

(f is one to one, $f(x^2) = (f(x))^2$, $f(x) = n$ for all $x \in [n-1, n)$, $f(x) = n$ for all $x \in [n, n+1)$).

(10) If $\lim\limits_{x \longrightarrow a} f(x) = f(a)$ and $\lim\limits_{x \longrightarrow a} \dfrac{f(x) - f(a)}{x - a} = \infty$ or $-\infty$ then the line $x = a$ is
(a vertical asymptote, a horizontal asymptote, a vertical tangent, a horizontal tangent).